

**Amendments to the Claims:**

This listing of claims will replace all prior versions, and listings of claims in the application:

**Listing of Claims:**

1. (Currently Amended) A computer-implemented method for computing the number of points on an elliptic curve ~~over a finite field, in which a Frobenius equation is solved to a given precision by first and second parts, wherein said parts comprise the following steps, the method comprising:~~

receiving an elliptic curve having a number of points on the curve; and

determining the number of points on the elliptic curve, wherein the determining includes solving a lifted Frobenius equation to a full precision by using first and second parts with a reduced precision, wherein the solving includes:

- a) computing, to the reduced precision, said first part firstly computes a first partial solution of said lifted Frobenius equation using said first part recursively ~~to reduced precision,~~
- b) ~~Said first part secondly applies~~ applying a Frobenius operation to said first partial solution,
- c) ~~Said first part thirdly computes~~ computing an error term for said lifted Frobenius equation,
- d) ~~Said first part fourthly computes~~ computing correction factors for said lifted Frobenius equation,
- e) ~~Said first part fifthly computes~~ computing, to the reduced precision, a second partial solution of a modified lifted Frobenius equation that includes the error term and the correction factors using said second part ~~to reduced precision, wherein computing the second partial solution includes:~~
- f) ~~Said first part sixthly combines said first partial solution and said second partial solution;~~

g) — ~~Said second part firstly computes~~ computing, to the reduced precision, a first third partial solution of said modified lifted Frobenius equation using said second part recursively ~~to reduced precision,~~

h) — ~~Said second part secondly applies~~ applying a Frobenius operation to said ~~first third~~ third partial solution,

i) — ~~Said second part thirdly updates~~ updating said error term,

j) — ~~Said second part fourthly computes~~ computing, to the reduced precision, a second fourth partial solution of said modified lifted Frobenius equation using said second part recursively ~~to reduced precision,~~

k) — ~~Said second part fifthly combines~~ combining said ~~first third~~ third partial solution and said ~~second fourth~~ fourth partial solution to create the second partial solution,-

f) ~~Said first part sixthly combines~~ combining said first partial solution and said second partial solution to provide the solution at the full precision,-

2. (Currently Amended) The method of claim 1 in which said reduced precision is one half of said ~~given~~ full precision.

3. (Original) The method of claim 1 in which said first and second parts compute the Teichmüller lift of a given finite-field polynomial.

4. (Original) The method of claim 1 in which said first and second parts compute the canonical lift of said elliptic curve.

5. (Original) The method of claim 1 in which said first and second parts compute the multiplicative representative of a given finite-field element.

6. (Original) The method of claim 1 in which said first and second parts compute the trace of a given p-adic number.

7. (Original) The method of claim 1 in which said first and second parts compute the norm of a given p-adic number.

8. (Currently Amended) The method of claim 4, further comprising:  
receiving a sequence of elliptic curves and determining the number of points on  
each elliptic curve, in which said first and second parts analyze a the sequence of elliptic curves.

9. (Currently Amended) The method of claim 8, further comprising:  
generating in which said analysis generates a cryptographic key for use in a  
digital processing system using one of the secure elliptic curves.

10. (New) The method of claim 1, further comprising:  
based on the number of points, identifying whether the elliptic curve is a secure  
elliptic curve for generating a cryptographic key.

11. (New) The method of claim 1, further comprising:  
storing the number of points on the elliptic curve in a memory of the computer.

12. (New) A computer readable medium embodying program code for  
directing one or more processors to perform an operation for computing the number of points on  
an elliptic curve, the operation comprising the steps of:

receiving an elliptic curve having a number of points on the curve; and

determining a number of points on the elliptic curve, wherein the determining  
includes solving a lifted Frobenius equation to a full precision by using first and second parts  
with a reduced precision, wherein the solving includes:

a) computing, to the reduced precision, a first partial solution of said lifted  
Frobenius equation using said first part recursively,

b) applying a Frobenius operation to said first partial solution,

c) computing an error term for said lifted Frobenius equation,

d) computing correction factors for said lifted Frobenius equation,

e) computing, to the reduced precision, a second partial solution of a  
modified lifted Frobenius equation that includes the error term and the correction factors using  
said second part, wherein computing the second partial solution includes:

computing, to the reduced precision, a third partial solution of said modified lifted Frobenius equation using said second part recursively,

applying a Frobenius operation to said third partial solution,  
updating said error term,

computing, to the reduced precision, a fourth partial solution of said modified lifted Frobenius equation using said second part recursively,

combining said third partial solution and said fourth partial solution to create the second partial solution,

f) combining said first partial solution and said second partial solution to provide the solution at the full precision.

13. (New) The computer readable medium of claim 12, wherein the operation further comprises the step of:

based on the number of points, identifying whether the elliptic curve is a secure elliptic curve for generating a cryptographic key.

14. (New) A integrated circuit configured to compute the number of points on an elliptic curve, the integrated circuit comprising:

logic for receiving an elliptic curve having a number of points on the curve;

logic for determining a number of points on the elliptic curve, wherein the determining includes solving a lifted Frobenius equation to a full precision by using first and second parts with a reduced precision, wherein the solving includes:

a) computing, to the reduced precision, a first partial solution of said lifted Frobenius equation using said first part recursively,

b) applying a Frobenius operation to said first partial solution,

c) computing an error term for said lifted Frobenius equation,

d) computing correction factors for said lifted Frobenius equation,

e) computing, to the reduced precision, a second partial solution of a modified lifted Frobenius equation that includes the error term and the correction factors using said second part, wherein computing the second partial solution includes:

computing, to the reduced precision, a third partial solution of said modified lifted Frobenius equation using said second part recursively,  
applying a Frobenius operation to said third partial solution,  
updating said error term,  
computing, to the reduced precision, a fourth partial solution of said modified lifted Frobenius equation using said second part recursively,  
combining said third partial solution and said fourth partial solution to create the second partial solution,  
f) combining said first partial solution and said second partial solution to provide the solution at the full precision.

15. (New) The integrated circuit of claim 14, further comprising:  
logic for identifying, based on the number of points, the elliptic curve as a secure elliptic curve for generating a cryptographic key.